Synopsis for Ph.D. Programme

Title of the Research:

A STUDY ON FUZZY CONE NORMED LINEAR SPACES AND FIXED POINT THEORY IN SUCH SPACES

Name of the candidate : Phurba Tamang, Department of Mathematics, Siksha-Bhavana, Visva Bharati, Santiniketan-731235

Name of the Guide: Dr. Tarapada Bag, Department of Mathematics, Siksha-Bhavana, Visva-Bharati, Santiniketan-731235

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Introduction:

The concept of fuzzy set was introduced by L.A. Zadeh in 1965. After that, the success of research in fuzzy set and fuzzy logic has been demonstrated in a variety of fields such as artificial intelligence, computer science, control engineering, computer applications, robotics and many more. Nowadays, one of the most important topics of fuzzy analysis is applying the results of classical analysis in fuzzy version. Especially many mathematicians tried to develop different notions of fuzzy structure. One such notion is of fuzzy norm which is defined from two different angles. One of them is due to A.K. Katsaras who defined fuzzy norm as an absorbing, balanced and convex fuzzy subset of a vector space. The other notion of fuzzy norm is introduced as a mapping from a vector space to the set of non-negative fuzzy real number. In 1975, Kramosil and Michalek introduced the concept of fuzzy metric space and studied some properties of such space. In 1984, Osmo Kaleva introduced the definition of fuzzy metric space in a different approach and proved Banach fixed point theorem in fuzzy setting. Later on, George and Veeramani gave a stronger form of metric fuzziness. In 1984, Katsaras first introduced the idea of fuzzy norm on a linear space. In 1992, Felbin introduced an idea of a fuzzy norm on a linear space whose associated metric is Kaleva type. In 1994, Cheng and Mordeson introduced an idea of a fuzzy norm on a linear space whose associated metric is Kramosil and Michalek type. Following Cheng and Mordeson, in 2003, Bag and Samanta introduced a definition of fuzzy norm and proved the decomposition theorem of fuzzy norm to a family of crisp norms.

On the other hand, a number of generalization in metric space, normed linear space and in inner product space have been done. Recently, Huang and Zhang introduced the notion of cone metric space and proved some fixed point theorems in cone metric space for mappings satisfying various contraction conditions. Several authors developed the cone metric space introduced by Huang and Zhang by establishing many fundamental results. Following the definition of cone metric space, Bag introduced the idea of fuzzy cone metric space as well as fuzzy cone normed linear space.

Let T be a self map of a nonempty set X. A point $p \in X$ such that $Tp = p$ is called a fixed point of the map T. In crisp setting, much work have been done in fixed point theory and it has also many applications in different field of mathematics such as differential equations, integral equations etc. The importance of the fixed point theory lies mainly in the fact that most of the equations arising in the various physical formulations may be transformed to fixed point equations or inclusions.

Our main aim is to develop functional analysis in fuzzy structure with an intention to introduce fuzzy cone normed linear space and study its properties. We also try to study fixed point theory on such spaces.
So far we have made some progress. Following the definition of fuzzy normed linear space introduced by Bag and Samanta, an idea of fuzzy cone normed linear space has been introduced and established some basic results.

Objectives

The main objectives are as follows:

i) To introduce the idea of fuzzy cone normed linear space which is a generalization of Bag and Samanta tybe fuzzy normed linear space and study its basic properties.

ii) To introduce the idea of fuzzy cone normed linear space which is a generalization of Felbin tybe fuzzy normed linear space and study its basic properties.

iii)To study the notion of operators on such spaces.

iv) To study the four fundamental results (Hahn Banach theorem, Uniform Boundedness Principle, Open mapping theorem, Closed graph theorem) on such spaces.

v) To study the fixed point theory on such spaces.

Work Done

i) P. Tamang, T. Bag, Some results on finite dimensional fuzzy cone normed linear space, Ann. Fuzzy Math. Inform.(accepted for publication).

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